An introduction to STGraph

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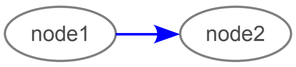
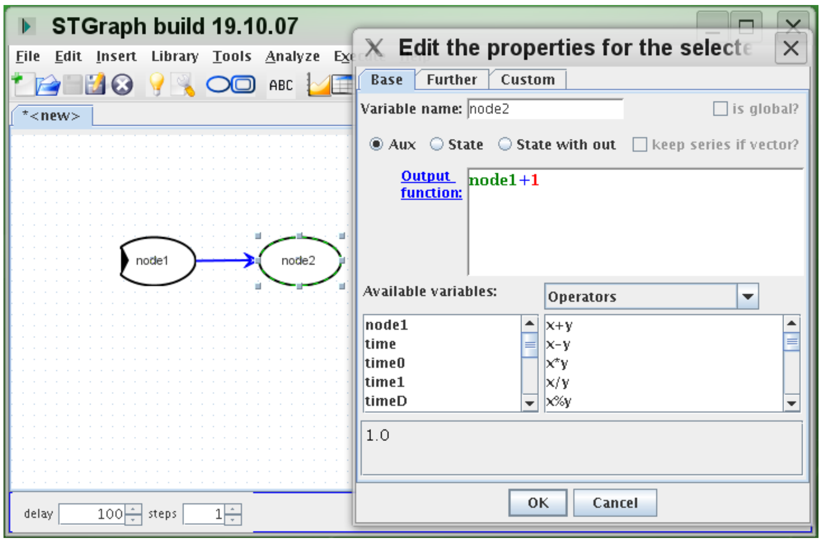
The latest version of STGraph and of its documentation can be freely downloaded from the site:
`http://www.liuc.it/persone/lmari/stgraph/`

STGraph is a simulator for dynamic systems modeled according to the state-variable approach to System Theory in the synchronous, discrete-time case.
Systems are qualitatively modeled in STGraph as *directed graphs*, whose nodes and arrows (i.e., directed edges) represent respectively variables and dependence relations in variable definition. For example, the simple graph:



is the model of a system characterized by two variables, called `node1` and `node2`, and such that the value of `node2` depends in its definition on `node1`.
Together with its *name*, each node is characterized by an *expression*, whose evaluation produces a *value* for the node. For example, if the expressions of `node1` and `node2` are `1` and `node1+1` respectively, then their values are evaluated as `1` and `2` respectively.
The main front-end of STGraph is a GUI interactive editor, which allows the user to create graphs with usual drag-and-drop techniques and to introduce and edit expressions in an interactive and contextual way.



Variable name: node2 ☐ is global?
[✓ Aux] State State with out ☐ keep series if vector?
Output function: node1+1
Available variables: node1, time, time0, time1, timeD
Operators: x+y, x-y, x*y, x/y, x%y
1.0

Expression Language

The language by which the expressions are defined, here called STEL (STgraph Expression Language), is a *functional* one, so that each node expression always produces one (and only one) value as result of its evaluation, no explicit variable assignment is allowed and no side effects can be generated in the evaluation. STEL is a *weakly typed* language (i.e., the variable type is automatically assigned by the system), with *double* as the basic data type, also interpreted as integer or boolean when required. Each variable can be:

- a *scalar*, written in decimal (e.g., `1.23`) or scientific (e.g., `1.23E2`) format; all strictly positive values are dealt with as the boolean 'true', and zero and negative values as the boolean 'false';
- a (column) *vector*, i.e., dynamic 1D array of scalars, written, e.g., as `[1,2,3]` to define the vector of the three scalars `1`, `2`, and `3`;
- a *matrix*, i.e., dynamic 2D array of scalars, written, e.g., as `[[1,2,3],[4,5,6]]` to define the

matrix of 2 rows and 3 columns whose element in first row and first column is the scalar `1`, the element in first row and second column is the scalar `2`, and so on.

STEL is a equipped with a fair amount of operators and functions [(*)], many of them behaving in a polymorphic way on scalars, vectors, and matrices. For example, if the expression defining `node2` is `node1+1`:

`node1` assigned value	`node1` type	`node2` computed value	`node2` type
`10`	scalar	`11`	scalar
`[10,20,30]`	vector	`[11,21,31]`	vector
`[[10,20],[30,40]]`	matrix	`[[11,21],[31,41]]`	matrix

and if the expression defining `node2` is `max(node1,20)`:

`node1` assigned value	`node1` type	`node2` computed value	`node2` type
`10`	scalar	`20`	scalar
`[10,20,30]`	vector	`[20,21,31]`	vector
`[[10,20],[30,40]]`	matrix	`[[20,21],[31,41]]`	matrix

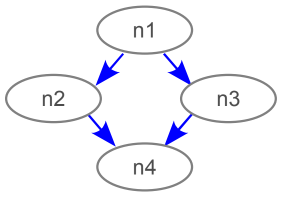
Together with several mathematical and statistical operators and functions, STEL includes two basic functions operating as control structures:

- a *conditional* structure, written `if(`c_1`,`v_1`,`c_2`,`v_2`,…,`c_n`,`v_n`,`v_{n+1}`)`, that returns v_1 if c_1 is true, else v_2 if c_{12} is true, else …, and v_{n+1} otherwise, thus operating as a chain of if … else if … else if … else …; in its simplest usage `if` has thus three parameters, `if(`c_1`,`v_1`,`v_2`)`: its value is v_1 if c_1 is true, and v_2 otherwise);
- an *iterative* structure, written `iter(x,y,z)`, that returns the value obtained by repeatedly applying the expression `y` to the elements of the vector `x`; the expression `y` can contain the system variables `$0`, that stores the partial result of the iteration and whose initial value is `z`, `$1`, that runs over the vector `x`, and `$i`, the index running over the same vector.

STEL also includes the meta-function `function()`, that allows the creation of user-defined functions.

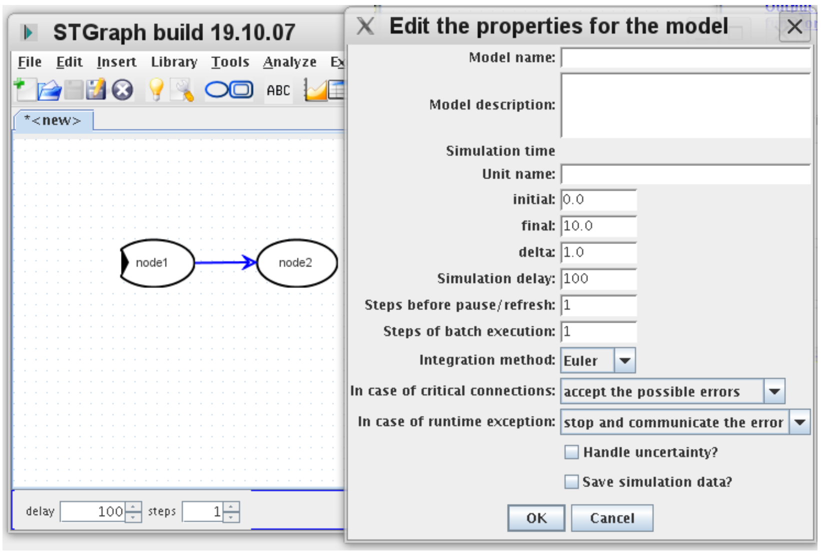
Logic of execution

Each node can be used to define an arbitrary number of other nodes, i.e., an arbitrary number of outgoing arrows can be drawn from each node. Conversely, each node can be defined by an arbitrary number of other nodes, i.e., an arbitrary number of incoming arrows can be drawn to each node. The network of the arrows in the graph implicitly defines the sequence of evaluation. Whenever the evaluation is not constrained, it is conceptually performed in parallel way. For example, in the simple graph:



(*) For a complete listing and documentation of the operators and functions in STEL, see “STGraph - System defined functions”, downloadable at the address http://www.liuc.it/persone/lmari/stgraph/app/functions.pdf
For the formal specification of the STEL grammar, see “STGraph - BNF for the Expression Language”, downloadable at the address http://www.liuc.it/persone/lmari/stgraph/app/Parser.html

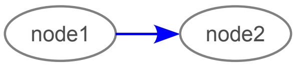
the sequence of evaluation is `n1`, then `n2` and `n3`, possibly evaluated in parallel, and finally `n4`. STGraph is aimed at simulating the behavior of dynamic systems. Hence, each model is characterized by a *time basis*, that must be set before executing the simulation. The related parameters are the initial time, `time0`, the final time, `time1`, and the time step, `timeD>0` for the hypothesis of time discreteness.



Edit the properties for the model

Model name:	
Model description:	
Simulation time	
Unit name:	
initial:	0.0
final:	10.0
delta:	1.0
Simulation delay:	100
Steps before pause/refresh:	1
Steps of batch execution:	1
Integration method:	Euler
In case of critical connections:	accept the possible errors
In case of runtime exception:	stop and communicate the error
	☐ Handle uncertainty?
	☐ Save simulation data?

When the user runs the simulation, the system-defined variable `time` subsequently assumes the values `time0`, `time0+timeD`, `time0+2timeD`, ..., `time1`, and the simulated behavior of the modeled system is traced by the sequence of values that each node assumes at these time steps. For example, assuming in the graph:



that the expressions of `node1` and `node2` are `time` and `node1+1` and that the time basis is such that `time0` is `0.0`, `time1` is `3.0`, and `timeD` is `1.0`, then the simulation trace is:

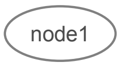
time	node 1	node 2
0	0	1
1	1	2
2	2	3
3	3	4

The assumption that the simulation is held in a synchronous way implies that the time basis is common to all the nodes, and therefore is a characteristic of the whole model.

Note types

Each node behaves as a basic computational unit, which transforms its (optional) inputs to an output, dynamically assigned as node value. The following *node types* are dealt with:

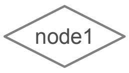
- *auxiliary nodes*, i.e., nodes whose output is synchronously determined as a function of their input only; auxiliary nodes are visualized as:



  if, for example, `node1` depends on `node0a` and `node0b`, then its output function could be:

```
node1-out := node0a+node0b
```

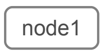
- *constant nodes*, specific cases of auxiliary nodes with no incoming arrows and whose defining expression is a constant (e.g., `1.0`) or an expression evaluated as a constant (e.g., `1.0+2.0`); constant nodes are visualized as:



hence, the behavior of a constant node named `node1` is fully characterized by its output function, e.g.:

```
    node1-out := 1.0+2.0
```

- *state nodes*, i.e., nodes whose output can be determined as a function of their input and their previous value; state nodes are visualized as:



hence, the behavior of a state node is actually determined by a state transition function (sometimes also called next state function), which computes the next state as a function of the current state (maintained by the system variable `this`) and the current value of the inputs, e.g.:

```
    node1-trans := 1.0+this
```

meaning that the next state (i.e., at the time instant `time+timeD`) will be `1.0` plus the current state; the value of a state node is equal to its current state, i.e., implicitly:

```
    node1-out := this
```

The definition of a state node requires its initial state be set, e.g.:

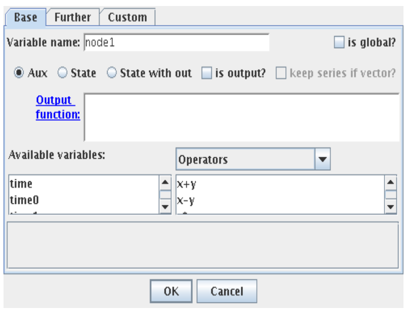
```
    node1-x0 := 0.0
```

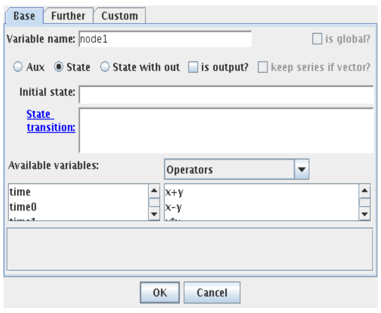
- *state nodes with distinct output*, i.e., state nodes whose output function is defined in explicit way, e.g.:

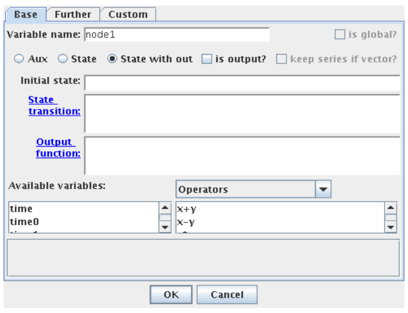
```
    node1-x0 := 0.0
    node1-trans := 1.0+this
    node1-out := this*2
```

meaning that the next state (i.e., at the time instant `time+timeD`) will be `1.0` plus the current state, starting from the value `0.0`, and the node value is equal to the current state times `2`; state nodes with distinct output are visualized as state nodes.

The type of a node and its defining expression(s) are set in the node configuration dialog.







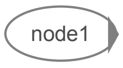
Auxiliary node:
defined by a behavior/output function

State node:
defined by an initial state
and a state transition function

State node with distinct output:
defined by an initial state
a state transition and an output function

Output and input nodes

In the node configuration dialog any node can be set to be an *output* node, visualized (in the example of an auxiliary node) as:



The time sequence of the values of an output node is maintained and can be visualized by means of an *output widget*, e.g., a chart or a data table.

The sequence is always maintained if the node value is scalar; in the case of vector values the history is maintained only if the “keep series if vectors?” switch has been set in the node configuration dialog; finally, in the case of matrix values the history is never maintained. Therefore:

node type	value	value sequence
scalar	current value (scalar)	sequence of values (vector)

vector	current value (vector)	if keep series:
		  sequence of values (matrix)
		else:
		  current value (vector)

matrix	current value (matrix)	current value (matrix)

Any auxiliary node without incoming arrows is automatically dealt with an *input* node, visualized (in the example of an auxiliary node) as:

The values of an input node can be set by means of an *input widget*, e.g., a slider or an input data table.

Graph topology

The node types also constrain the graph topology, and specifically the possibility of introducing loops (acyclic graphs are always topologically correct), according to the following general rule:

- any loop must include at least one state node

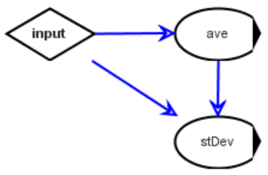
also taking into account that from the point of view of this rule state nodes with distinct output are considered equivalent to auxiliary nodes.

Submodels

Together with the listed node types, STGraph includes the *submodel* node type, by which each previously generated model can be embedded as a submodel in a model. An embedded submodel is accessible to its supermodel only through its input and output nodes, according to the following logic:

- the input nodes of a submodel are automatically exposed to the supermodel via the submodel node configuration dialog. Hence, the value of each input node of a submodel can be either left unassigned, and in this case its value is computed by its output function, or can be assigned by the supermodel;
- the output nodes of a submodel are automatically exposed to the supermodel as multiple node outputs, each of them accessible with the syntax `submodelnode.outputnode`.

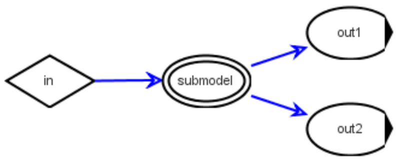
For example, the simple model:



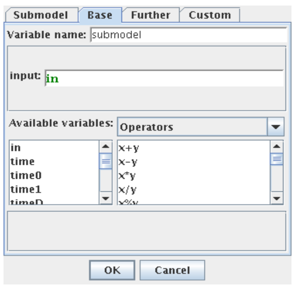
with the following definitions for the nodes:

```
input-out := [1:5] // just the example of a vector
ave-out := if(!isVector(input)||size(input)==0,
                0,
                iter(input,$0+$1,0)/size(input)
           ) // compute the arithmetic mean of the input vector elements
stDev-out := if(!isVector(input)||size(input)==0,
                0,
                sqrt(iter((input-ave)^2,$0+$1,0)/(size(input)-1))
           ) // compute the standard deviation of the input vector elements
```

can be embedded as a submodel in the following model:



The configuration dialog for the `submodel` node could be:



Variable name: submodel

input: in

Available variables: Operators

while the output nodes in the supermodel are simply defined as:

```
    out1-out = submodel.ave
    out2-out = submodel.stDev
```

STGraph and System Theory

STGraph is aimed at giving an implementation as pure as possible of the state-variable approach to System Theory in the synchronous, discrete-time case, in which each model Σ is formalized by means of a 7-tuple $\Sigma = \langle T, U, \Omega, Y, X, \varphi_k, \eta_k \rangle$ such that:

- the *time basis* T in which the system dynamics is analyzed; T can be an interval of real numbers, $T = [t_0, t_1]$, in case of continuous time, or a sequence $T = \{t_{0,} t_{1,} \ldots, t_n\}$, in case of discrete time; in the latter case, the time points t_i are usually assumed to be regularly spaced, $t_{i+1} = t_i + \Delta t$, and the time basis is simply defined by the start time, t_0, the end time, t_n, and the time step Δt;
  - ✔ STGraph implements the time base in the discrete time case with constant time step, by specifying the start time, `time0`, the time step, `timeD`, and the end time, `time1`;
- the *input set* $U = \{u_i\}$ of the values u_i that can come from the environment to the system; the algebraic structure of this set is not a priori constrained, and in particular can be either continuous or discrete;
  - ✔ STGraph implements the input set as the Cartesian product of the sets of the values of all the input variables, i.e., the auxiliary variables without incoming arrows;
- the *input function* $u : T \rightarrow U$, that in each instant $t \in T$ expresses the input $u = u(t)$, $u \in U$ coming to the system; in the general case, the input function could be not analytically determined, and then the set of the *admissible input functions* $\Omega = \{u_j\}$ must be stated;
  - ✔ STGraph implements the input function as the expressions that define the value of the input variables; this definition operatively specifies the set Ω;
- the *output set* $Y = \{y_i\}$ of the values y_i that are designed to be observable; the algebraic structure of this set is not a priori constrained, and in particular can be either continuous or discrete;
  - ✔ STGraph implements the output set as the Cartesian product of the sets of the values of all the output variables, i.e., the auxiliary variables whose output attribute has been set;
- the *state set* X of the system, and in particular the initial state $x(t_0)$; the algebraic structure of this set is not a priori constrained, and in particular can be either continuous or discrete;
  - ✔ STGraph implements the state set as the Cartesian product of the sets of the values of all the state variables;
- the *state transition function* $\varphi : U \times X \rightarrow X$, that in the case of discrete time becomes $x(t + \Delta t) = \varphi(u(t), x(t))$;

    - ✔ STGraph implements the state transition function as the `node-trans` expressions that define the next state of the state variables, together with the `node-x0` expressions that define the initial state of such variables;
- the *behavior function* $\eta : U \times X \rightarrow Y$, that in the case of discrete time becomes $y(t)=\eta(u(t), x(t))$, being $y : T \rightarrow U$ the *output function* of the system (note that usually the term “output function” is adopted also for behavior functions);
    - ✔ STGraph implements the behavior function by means as the `node-out` expressions that define the value of the variables.